Solution to Problem 67

The most probable value is given by a least squares solution. Let \( L_1 = AB, \ L_2 = BC, \ L_3 = CD \) with residuals \( r_1, r_2 \) and \( r_3 \), respectively.

Set up condition equations:
\[
\begin{align*}
AB + r_1 &= L_1, \quad \text{so} \quad r_1 = L_1 - AB = L_1 - 49.897 \\
BC + r_2 &= L_2, \quad \text{so} \quad r_2 = L_2 - BC = L_2 - 99.846 \\
CD + r_3 &= L_3, \quad \text{so} \quad r_3 = L_3 - CD = L_3 - 99.931 \\
AC + r_1 + r_2 &= L_1 + L_2, \quad \text{so} \quad r_1 + r_2 = L_1 + L_2 - AC = L_1 + L_2 - 149.797 \\
BD + r_2 + r_3 &= L_2 + L_3, \quad \text{so} \quad r_2 + r_3 = L_2 + L_3 - BD = L_2 + L_3 - 199.812
\end{align*}
\]

To get a least squares solution the sum of the squares of the residuals must be minimized, to wit:
\[
\phi = r_1^2 + r_2^2 + r_3^2 + (r_1 + r_2)^2 + (r_2 + r_3)^2 = (L_1 - 49.897)^2 + (L_2 - 99.846)^2 + (L_3 - 99.931)^2 + (L_1 + L_2 - 149.797)^2 + (L_2 + L_3 - 199.812)^2
\]
must be minimized.

The partial derivative evaluated with respect to each estimate and equated to zero will minimize \( \phi \):
\[
\frac{\partial \phi}{\partial L_1} = 2( L_1 - 49.897 ) + 2( L_1 + L_2 - 149.797 ) = 0
\]
\[
\frac{\partial \phi}{\partial L_2} = 2( L_2 - 99.846 ) + 2( L_1 + L_2 - 149.797 ) = 0
\]
\[
\frac{\partial \phi}{\partial L_3} = 2( L_3 - 99.931 ) + 2( L_2 + L_3 - 199.812 ) = 0
\]

\[
\begin{align*}
L_1 - 49.897 + L_1 + L_2 - 149.797 &= 0, \quad \text{or} \quad 2L_1 + L_2 = 199.694 \\
L_2 - 99.846 + L_1 + L_2 - 149.797 + L_2 + L_3 - 199.812 &= 0, \quad \text{or} \quad L_1 + 3L_2 + L_3 = 499.455 \\
L_3 - 99.931 + L_2 + L_3 - 199.812 &= 0, \quad \text{or} \quad L_2 + 2L_3 = 299.743 \\
2L_1 + 6L_2 + 2L_3 - 2L_1 - L_2 &= 699.216 = 5L_2 + 2L_3 \\
\text{but} \quad 5L_2 + 10L_3 &= 1498.715, \quad \text{so} \quad 8L_3 = 799.499 \text{ and } L_3 = 99.9374 \\
\text{From which } L_1 &= 49.9129, \ L_2 = 99.8683 \text{ and } A-D = 249.7185
\end{align*}
\]
Solution to Problem 68

"OPEN" THE BOX SO IT LAYS FLAT. THERE ARE TWO POSSIBILITIES.

DOWN THE WALLS AND ACROSS THE FLOOR:

UP THE WALLS AND ACROSS THE CEILING:

THE CEILING ROUTE IS SHORTER