Solution to Problem 55

\[ AB = Aa \times \tan(90° - 0° 14'39.5") \]

\[ Aa = 1 \text{ meter} \]

\[ AB = \tan 89° 45'20.5" = 234.52 \text{ meters} \]
Solution to Problem 56

SOLUTION TO PROBLEM NO. 56

\[ \frac{\alpha}{2} \]

The distance AB is also $Aa \times \cot \frac{\alpha}{2}$, where $\alpha$ is the measured angle.

Differentiating $AB$ with respect to $\alpha$,

\[ dAB = -(Aa)(\frac{1}{2})(\csc^2 \frac{\alpha}{2}) \, d\alpha \]

$Aa = 1$ and $\csc^2(\frac{\alpha}{2}) = \frac{1}{\sin^2(\frac{\alpha}{2})}$

\[ \frac{1}{\pm0.0001 \, AB} = \frac{-d\alpha}{2 \sin^2(\frac{\alpha}{2})}, \text{ where } d\alpha \text{ is } 0.0000048481368 \text{ radians} \]

\[ \cot(\frac{\alpha}{2}) \] \[ \frac{1}{\pm0,0000048481368} = \frac{\cot(\frac{\alpha}{2})}{2 \sin^2(\frac{\alpha}{2})} \]

\[ \sin^2(\frac{\alpha}{2}) \cot(\frac{\alpha}{2}) = -/+/0.024240684 \text{ (if } \alpha \text{ is too large by one second, } AB \text{ will be too small, and vice versa)} \]

\[ \sin(\frac{\alpha}{2}) \cos(\frac{\alpha}{2}) = -/+/0.024240684 = \frac{\sin \alpha}{2} \]

from which $\alpha = 0.024243059 \text{ radians or } 1^\circ23'20.5''$

and $AB = 41.24 \text{ metres}$