Solution to Problem 43

SOLUTION TO PROBLEM NO. 43 (TEXT)
Let the sea level distance be equal to the measured distance, \( m \), less some correction, \( c \).
Let \( R \) equal the radius of the earth at sea level and let \( a \) equal the average height.

\[
\begin{align*}
    m - c &= m \\
    \text{By similar sectors:} & \quad \frac{m}{R} = \frac{c}{R + a} \\
    \frac{m}{R} + \frac{c}{R + a} &= \frac{m}{R} (\frac{1}{R} + \frac{a}{R^2} + ... ) \\
    \frac{ma}{R} + \frac{ma^2}{R^2} + \frac{ma^3}{R^3} &= m + c \\
    \frac{c}{R^2} + \frac{c}{R^3} &= 2.19537' - 0.00045' + 0.0000001'... = 2.195' \\
    \text{sea level} &= 10,643.87' - 2.195' = 10,641.675' \\
    \text{OR}, & \quad m = s = 20,906,000 \\
    \frac{m}{R} = \frac{s}{R + a} = 10,643.87' = \frac{20,906,000}{R + a} \\
    \text{Elevation differences become significant when } c = 0.005' \text{ or } \frac{a}{R + a} = 0.99995 \\
    100 a \\
    \frac{c}{R + a} = 0.005' = \frac{0.99995}{20,906,000} \text{, that is when } a = 1045' \\
    \text{OR}, & \quad R = 20,906,000 \\
    \frac{R}{R + a} = 0.99995 \text{, } R - 0.99995 R = 0.99995 a \text{, } a = 1045' \\
    \text{Note: } R = 20,906,000' \text{ per Surveying by Bouchard & Moffit, Ninth edition, 1992, page 375.} \\
\end{align*}
\]
**Solution to Problem 44**

SOLUTION TO PROBLEM NO. 44 (TEXT)

Let the highway elevation = 100.00'. The inner rail elevation is therefore 103.00'.
The outer rail elevation is 103.00' plus the super-elevation:

\[
e \text{ (feet)} = \frac{0.0026585 \times 80^2 \times (1.2)}{12} = 0.44', \text{ so the outer rail elevation is 103.44'.}
\]

The grade across the tracks is 0.44'/5' = 0.088, or 8.8%. In Figure 1, a change from 0% grade to +8.8% grade is

\[
\frac{8.8 - 0}{0.682} = 12.9, \text{ but a comfortable } r = \frac{15,000}{70^2} = 3.06.
\]

The transition must be made as shown in Figure 2.
The rate for \( L_1 \) cannot exceed \( r = -3.06 \) and \( L_2 \) cannot exceed \( r = +3.06 \)

\[
r_1 = -3.06 = \frac{g_1 - 0}{L_1} \quad \text{and} \quad r_2 = 3.06 = \frac{8.8 - g_1}{L_2}
\]

so \( g_1 = -3.06 \times L_1 \) and \( L_2 = L_1 + 2.8758 \) (substituting \( g_1 \) in first equation into second)

also, 100.00' + \( g_1 \) \(( L_1 / 2 + L_2 / 2) + 8.8 \times (L_2 / 2) = 103.00'

substituting \(-3.06 \times L_1 \) for \( g_1 \) and \(( L_1 + 2.8758 \) for \( L_2 \) then expanding and rearranging yields

\[
L_1 = 1.7762, \ g_1 = -5.435%, \text{ and } L_2 = 4.6520
\]

Likewise, \( r_3 \) cannot exceed \(-3.06\) and \( r_4 \) cannot exceed \(+3.06\)

\[
r_3 = -3.06 = \frac{g_2 - 8.8}{L_3}, \text{ and } r_4 = 3.06 = \frac{0 - g_2}{L_4}
\]

so \( g_2 = -3.06 \times L_4 \) and \( L_3 = L_4 + 2.8758 \)

and, 103.44' + 8.8 \(( L_3 / 2 + L_4 / 2) + g_2 \times (L_3 / 2 + L_4 / 2) = 100.00'

again, substituting \(-3.06 \times L_4 \) for \( g_2 \) and \(( L_4 + 2.8758\) for \( L_3\), expanding and rearranging,

\[
L_4 = 2.2933, \ L_3 = 5.1691 \text{ and } g_2 = -7.0175%
\]