Solution

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Divide the irregular pentagon ABCDE into quadrilateral BCDE and triangle ABE.

The radius of the circumscribing circle of a triangle in terms of its sides is

$$R = \frac{a \cdot b \cdot c}{4\sqrt{s(s-a)(s-b)(s-c)}}, \text{ where } s = \frac{a+b+c}{2} \text{ .... [1]}$$

The radius of the circumscribing circle of a cyclic quadrilateral can be found in terms of its sides by

$$(4R)^2 = \frac{(a \cdot b + c \cdot d)(a \cdot c + b \cdot d)(a \cdot d + b \cdot c)}{s(s-a)(s-b)(s-c)(s-d)}, \text{ where } s = \frac{a+b+c+d}{2} \text{ .... [2]}$$

Divide by 100 to simplify calculations, call line BE=x and equate [1] and [2]:

$$\frac{3 \cdot 7 \cdot x}{4\sqrt{\left(\frac{10+x}{2}\right)\left(\frac{10+x}{2}-3\right)\left(\frac{10+x}{2}-7\right)\left(\frac{10+x}{2}-x\right)}} =$$

$$\frac{\sqrt{(4 \cdot 5 + 6x)(4 \cdot 6 + 5x)(4x + 5 \cdot 6)}}{4\sqrt{\left(\frac{15+x}{2}-4\right)\left(\frac{15+x}{2}-5\right)\left(\frac{15+x}{2}-6\right)\left(\frac{15+x}{2}-x\right)}},$$

$$\frac{21x}{\sqrt{(40+14x+x^2)(-40+14x-x^2)}} = \frac{\sqrt{(480+244x+30x^2)(4x+30)}}{\sqrt{(35+12x+x^2)(45+12x-x^2)}},$$

$$\frac{441x^2}{-x^4+116x-1,600} = \frac{120x^3+1,876x^2+9,240x+14,400}{-x^4+154x^2+960x+1,575} \text{ from which}$$

$$120x^7 + 1,435x^6 - 4,680x^5 - 135,302x^4 - 456,480x^3$$
$$+2,025,775x^2 + 14,784,000x + 23,040,000 = 0$$

This can be set up in "Goal Seek" in Excel or solved by any good on-line equation solver. If using goal seek, start with a seed of 8, 9, or 10 to get x = 8.348344255. If you start with 4, 5, or 6 you will get x = 5.50633893, but x is obviously larger than 7. There are, after all, seven solutions to this equation.

Use [1] to solve:

$$R = \frac{3 \cdot 7 \cdot 8.348344255}{4\sqrt{(9.1741721275)(6.1741721275)(2.1741721275)(.8258278725)}} = 4.346054,$$

or R= 434.605 after multiplying by 100.