Solution

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Let AB=a, BC=x, CD=b, angle AEB=P, angle AEC=Q and angle AED=R

$$\frac{BE}{\sin A} = \frac{a}{\sin P} \text{ and } \frac{CE}{\sin A} = \frac{a+x}{\sin Q}\text{, or } \frac{BE}{a} = \frac{\sin A}{\sin P} \text{ and } \frac{CE}{a+x} = \frac{\sin A}{\sin Q}$$

dividing the first by the second, term for term,

$$\frac{\frac{BE}{CE}}{\frac{a}{a+x}} = \frac{\frac{\sin A}{\sin A}}{\frac{\sin P}{\sin Q}} \text{ so } \frac{BC}{CE} = \frac{\frac{a}{a+x}}{\frac{\sin P}{\sin Q}} = \frac{a \cdot \sin Q}{(a+x)\cdot \sin P} \quad \text{.. [1]}$$

$$\frac{BE}{\sin(A+R)} = \frac{x+b}{\sin(R-P)} \text{ and } \frac{CE}{\sin(A+R)} = \frac{b}{\sin(R-Q)}$$

$$\text{or } \frac{BE}{x+b} = \frac{\sin(A+R)}{\sin(R-P)} \text{ and } \frac{CE}{b} = \frac{\sin(A+R)}{\sin(R-Q)}$$

again dividing and rearranging, $\frac{BC}{CE} = \frac{(x+b)\cdot\sin(R-Q)}{b\cdot\sin(R-P)}$ .... [2]

equating [1] and [2]:

$$\frac{a\cdot\sin Q}{(a+x)\cdot\sin P} = \frac{(x+b)\cdot\sin(R-Q)}{b\cdot\sin(R-P)} \text{ and}$$

$$\frac{a\cdot b\cdot\sin Q\cdot\sin(R-P)}{\sin P\cdot\sin(R-Q)} = (a+x)(b+x) = ab + (a+b)\cdot x + x^2$$

All of the terms on the left are known values:

$$\frac{(505.677)(644.493)(\sin 69^\circ 35'30'')(\sin 84^\circ 53'34'')}{(\sin 17^\circ 23'04'')(\sin 32^\circ 41'08'')}$$
$$= (505.677)(644.493) + (505.677 + 644.493)\cdot x + x^2$$

from which $x^2 + 1{,}150.17x - 1{,}559{,}653.71 = 0$

and by the quadratic equation, $x = \frac{-1{,}150.17 \pm \sqrt{1{,}150.17^2 - 4(-1{,}559{,}653.71)}}{2}$

x = 799.825