Solution 243

by Benjamin Bloch, Ph.D.

For the circle, radius r:
Area = πr^2; Perimeter = $P_1 = 2\pi r$

Equilateral triangle, side l:
Area = $1/4\ l^2\ (3)^{1/2} = 0.433\ l^2$; Perimeter = $P_3 = 3l$

Square, side s:
Area = s^2; Perimeter = $P_4 = 4s$

Regular convex pentagon, side d:
Area = $5/4\ d^2 \tan 54 = 1.720\ d^2$; Perimeter = $P_5 = 5d$

Because all areas are equal we get the relationships between all of the sides and the circle radius, namely $\pi r^2 = 0.433\ l^2 = s^2 = 1.720 d^2$
Letting t_1, t_3, t_4, and t_5 equal the times required to walk P_1, P_3, P_4 and P_5 respectively, and because the rate of walking is uniform:
$P_1/t_1 = P_3/t_3 = P_4/t_4 = P_5/t_5$
In this problem, $t_1 = 2$ hours

1. We can now solve for l in terms of r
$l = (\pi/0.433)^{1/2}\ r = 2.694r$, so that $P_3 = 3l = 8.081r = 8.081P_1/2\pi = 1.286P_1$
The perimeter of the triangle is 1.286 times the circumference of the circle.
From before, $P_1/t_1 = P_3/t_3$ so that $t_3 = P_3t_1/P_1 = 1.286 \times 2$ hours = 2.57 hours to walk the triangular shaped plot.

2. Solving for s in terms of r, and then substitute this value in P_4
$P_4 = 4s = 4r(\pi)^{1/2}$ and $r = P_1/2\pi$, so that $P_4 = 2P_1 / (\pi)^{1/2} = 1.128\ P_1$
The perimeter of the square is 1.128 times the circumference of the circle.
From before, $P_1/t_1 = P_4/t_4$ so that $t_4 = P_4t_1/P_1 = 1.128 \times 2$hours = 2.26 hours to walk the square shaped plot.

3. Solving for d in terms of r and substituting into P_5 yields
$d = 1.351r$ so that $P_5 = 5d = 6.757r = 6.757P_1/2\pi = 1.075P_1$

Thus, it takes 1.075 x 2 hours = 2.15 hours to walk the pentagon perimeter.