

**Solution**  
**233**

by Benjamin Bloch, Ph.D.

Let  $a = \text{angle HFP}$ ,  $b = \text{angle GFH}$ , and  $c = \text{angle AFG}$

Then  $a = b + c$ , and  $a + b + c = 2a$

In triangle HFP:  $FP = 0.382 / \tan a$

In triangle AFP:  $FP = 1 / \tan 2a$

Therefore  $0.382 / \tan a = 1 / \tan 2a$

Since  $\tan 2a = 2 \tan a / (1 - \tan^2 a)$

We find that  $\tan^2 a = 0.236$ , and thus  $a = 25.91$  degrees

Therefore  $FP = 1 / \tan 2a = 0.786$

Angle FHP =  $90 - a = 64.1$  degrees

Now angle FAP =  $90 - 2a = 38.2$  degrees and angle FHG =

$180 - 64.1 = 115.9$  degrees

$GH = 1 - 2 \times 0.382 = 0.236$  and therefore  $GP = 0.618$

(Line AP is subdivided into the Golden Proportion; see my guest editorial.)

In triangle GFP:  $\tan(a + b) = 0.618 / 0.786 = 0.786$

Therefore  $a + b = 38.167$  degrees, making  $b = 12.26$  degrees

Since  $b + c = a$ , or  $12.26 + c = 25.91$ , we see that  $c = 13.65$  degrees

Angle FGP =  $90 - (a + b) = 90 - 38.167 = 51.83$  degrees (the famous Great Pyramid Angle)

Finally, angle FGA =  $180 - 51.83 = 128.17$  degrees

This gives us all of the angles.

Now  $FA = 1 / \sin 2a = 1.272$

$FG = 0.618 / \sin(a + b) = 0.618 / \sin 38.167 = 1$

Finally,  $FH = 0.382 / \sin a = 0.874$

