Solution

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Triangle ACB is isosceles whose equal base angles add to 160 degrees, so that angle ACB = 180 - 160 = 20 degrees.

Triangle BDC is thus isosceles, with equal base angles of 20 degrees each, therefore DB = DC.

In triangle AEB the sum of the base angles is 150 degrees so that angle AEB must be 180 - 150 = 30 degrees.

In triangle ADB the sum of the base angles is 140 degrees so that angle ADB must be 180 - 140 = 40 degrees.

Now angle CED + x + 30 = 180 degrees (a straight angle) so that angle CED must equal 150 - x.

The Law of Sines relates to the ratio of a triangle side to the sine of its opposite angle.

1. In triangle ADB: (DB/sin 80) = (AD/sin 60)
2. In triangle ADE: (AD/sin x) = (DE/sin 10)
3. In triangle DCE: (DC/sin(150 - x)) = (DE/sin 20) which becomes, since DB = DC: (DB/sin(150 - x)) = (DE/sin 20)

These last three steps using the Law of Signs are three independent simultaneous equations in three unknown distances: DB, AD, and DE. We can use these to get two expressions for DE, which we set equal to one another.

DE = (DB sin 20)/sin(150 - x) = ((DB sin 60) sin 10)/ (sin 80)(sin 10)

Dividing both sides by DB, we can solve for x.

After much manipulating we finally get: tan x = 0.364, and x = 20 degrees.