Solution
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by David Lindell, LS

Choose point P on line AQ so QP=QC

Angle AQC = 60° because it subtends the same chord as angle ABC.

With angle PQC = 60° and QP = QC, triangle PQC is equilateral.

Angle APC = 180° – 60° = 120°, so angle BQC = 120°
(again, subtending equal chords, or angle BQC = ½ arc BAC = ½ · 120°)

Triangles APC and BQC are therefore congruent, with AP = BQ and PC = QC

But AP + PQ = AQ so AQ = BQ + QC

From the Law of Cosines:
$BC^2 = 612.884^2 = 457.697^2 + BQ^2 - 2 \cdot BQ \cdot 457.697 \cdot \cos 120°$

$BQ^2 + 457.697 \cdot BQ - 166{,}140.254 = 0$

$$BQ = \frac{-457.697 \pm \sqrt{457.697^2 - (4)(-166{,}140.254)}}{2}$$

From which BQ = 238.604

AQ = 238.604 + 457.697 = 696.301

In triangle BRS, angle SBR = 30° and SB = ½ (612.884) = 306.442

BR, the radius = 306.442 ÷ cos30° = 353.849