

Solution
222

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The ratio 3:4:5 implies a total of $3+4+5 = 12$ parts, or $3/12$, $4/12$ & $5/12$.

If we let the radius equal unity, the total area is $\pi = 3.14159265$, making the parts $3\pi/12$, $4\pi/12$ and $5\pi/12$, or 0.78539816 , 1.04719755 and 1.30899694 .

The area of a segment is $\frac{1}{2} R^2(\phi - \sin\phi)$, where ϕ is the central angle in radians.

For the smaller area, $\phi = \text{angle AQB}$ and $0.78539816 = \frac{1}{2}(\phi - \sin\phi)$. Since the sine function varies from 0 to 1, ϕ cannot exceed $(2)(0.78539816)+1 = 2.57079632$ radians or $147^\circ 17' 45''$. Trying 140° (2.4434610 radians) gives an area of 0.900337 ; 130° (2.268928 radians) gives an area of 0.751442 , which brackets the area we want. Linear interpolation between these two values yields 2.308731 radians ($132^\circ 16' 50''$), but reinserting 2.308731 into the equation yields an area of 0.784436 . Using another value close to this, say $132^\circ 30'$ or 2.312561 radians, we get an area of 0.787642 . Interpolating in this small range gives 2.309880 radians ($132^\circ 20' 47''$), yielding an area of 0.785397 . [$1''$ more yields 0.785401 and $1''$ less yields 0.785393]
QB and AQ are $\frac{1}{2}(180^\circ - 132^\circ 20' 47'') = 23^\circ 49' 36.5''$ above the "equator", EE'. The chord AB is 1.829542 and the apothem is 0.403973 .

By the same logic for the larger area, $\phi = \text{angle CQD}$ and $1.30899694 = \frac{1}{2}(\phi - \sin\phi)$. But for $\phi = 180^\circ$, the area is $\pi/2 = 1.570796$. Interpolating between 140° (2.443461 radians) with an area of 0.900337 and 180° with an area of 1.570796 , yields a value near 2.868988 radians ($164^\circ 22' 51''$), for an area of 1.299872 when reentered into the equation. Utilizing a value close to this, say 165° (2.879793 radians) gives an area of 1.310487 . Interpolating within this small bracket yields 2.878276 radians ($164^\circ 54' 47''$), which gives an area of 1.30899565 .
QC and QD are $\frac{1}{2}(180^\circ - 164^\circ 54' 47'') = 7^\circ 32' 36.5''$ below the "equator", EE'. The chord CD is 1.982691 and the apothem is 0.131278 .

The center section is equal to $(2)(\frac{1}{2})(1^2)(23^\circ 49' 36.5''_{\text{rad}} + 7^\circ 32' 36.5''_{\text{rad}}) + \frac{1}{2}(1.829542)(0.403973) + \frac{1}{2}(1.982691)(0.131278) = 1.047199$

