Solution
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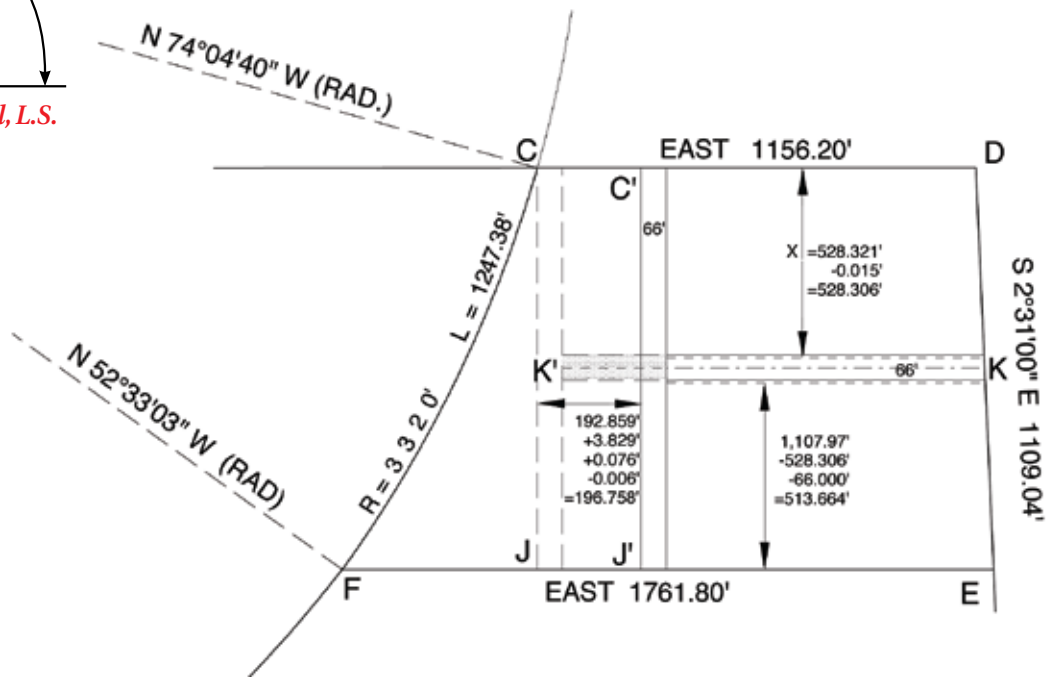

CDEF is 36 acres = 1,568,160 sq. ft., the perpendicular distance between the north and south lines is 1,107.97' and the segment area of arc CF is 48,373.60 sq. ft. Rotate 1°46'04" counterclockwise to make CD and FE "East". Drop a perpendicular from C to line FE at point J. FJ=556.903 and JE=1,204.897.

The area of CFJ is $\frac{(556.903)(1{,}107.97)}{2} - 48{,}373.60 = 260{,}142.31\ \text{ft}^2$

Let CJ be the west side of the north-south road and let the center of the east-west road, K-K', be the midpoint of DE. The area of the north-south road is 1,107.97×66=73,126.02 ft². The area of the east-west road is $\left(\frac{1{,}156.20 - 66 + 1{,}204.897 - 66}{2}\right)\cdot 66 = 73{,}560.20\ \text{ft}^2$.

The total area available is then 1,568,160 – 73,126.02 – 73,560.20 = 1,421,473.68 ft², or 473,824.59 ft² each.
Kelly gets 473,824.59 ft² – 260,142.31 ft² = 213,682.28 ft² more. 213,682.28/1,107.97 = 192.859, the amount to move line CJ east to C'J'. But doing so eliminates 192.859×66 =12,728.69 ft² of the east-west road, giving 12,728.69/3 = 4,242.90 ft² additional area to each. Moving line C'J' an additional 4,242.90/1,107.97 = 3.829 ft east eliminates 3.829×66=252.71 ft² of road, giving another 252.71/3 = 84.24 ft² to each. One more move of line C'J' 84.24/1,107.97 or 0.076 east gives a total of 192.859+3.829+0.076 = 196.764, adding 196.764×1,107.97 = 218,008.61 ft² to the area CFJ for a total of 478,150.92 ft² for Kelly.
Kathy, Karen and the east-west road now have 1,568,160 – 478,150.92 – 1,107.97x66 = 1,016,883.06 ft² total.

The road area is $\left(\frac{893.436 + 942.133}{2}\right)\times 66 = 60{,}573.78\,\text{ft}^2$, leaving 478,154.64 ft² each for Kathy and Karen. For Kathy to have 478,154.64 ft², $478{,}154.64 = \left(\frac{893.436 + x\cdot\tan 2°31' + 893.436}{2}\right)\cdot x$, from which x=528.321

Because the east-west road moves south 7.336 ft, it adds 21.28 ft² to its area (the length increases 7.336×tan2°31' and it is 66 ft. wide), lessening each area by 7.09 ft².

Moving line C'J' 0.006 west decreases Kelly's area by 0.006×1,107.97 = 6.65 ft² to 478,144.27 ft². The centerline of the east-west road is now 893.442 + (528.321 + 33)tan 2°31' = 918.113 and its area is 66×918.113 = 60,595.46 ft²

Kathy now has ½(893.442 + 528.321xtan2°31' + 893.442)x528.321 = 478,158.23 ft²
and Karen has ½ (942.139 – 513.649xtan2°31' + 942.139)× 513.649 = 478,130.66 ft² , the difference being corrected by moving line K-K' north 0.015', so that Kathy has ½(893.442 + 528.306×tan2°31' + 893.442)× 528.306 = 478,144.48 ft²
and Karen has ½(942.139 – 513.664xtan2°31' +942.139)× 513.664 = 478,144.45 ft²

Kelly's lot is 196.76' along the north, 753.66' along the south and 1,107.97' along the east. Kathy's lot is 893.44' along the north, 916.66' along the south, 528.31' on the west and 528.82' on the east. Karen's lot is 942.14' on the south, 513.66' on the west, 919.56' on the north and 514.16' on the east.