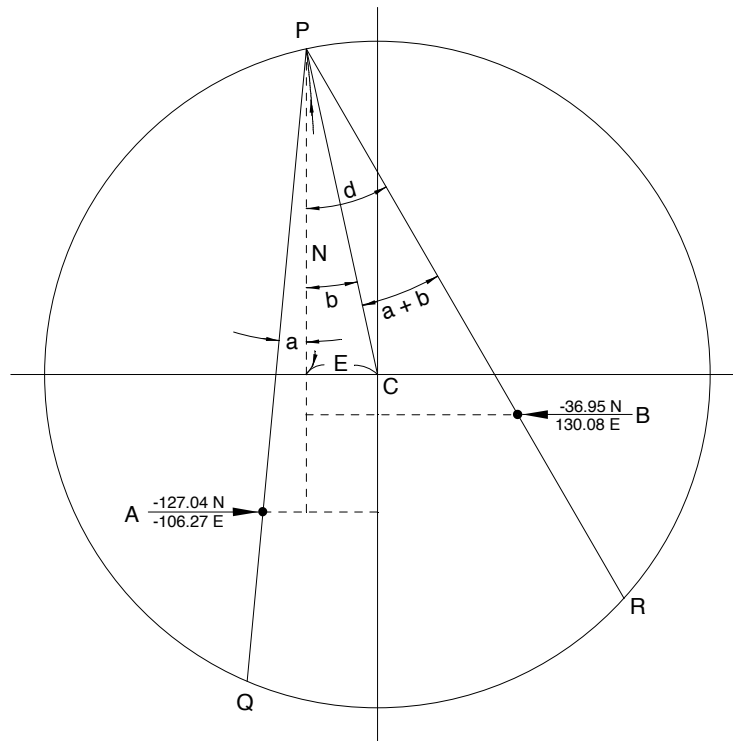


Solution
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Let the coordinates of point P be N & E, as shown. (Note that E is negative.)

Let angles a, b and d be as shown and let r be the radius of the circle, 308.649.

When line PQ equals line PR, line PC will be the bisector of angle QPR, or a+b.

$$(d + a) = 2(a + b), \text{ or } d - a = 2b$$

$$\tan(d - a) = \tan 2b, \tan d = \frac{130.08 - E}{N + 36.95}, \tan a = \frac{E + 106.27}{N + 127.04}, \text{ and } \tan b = \frac{E}{N}$$

$$\frac{\tan d - \tan a}{1 + \tan d \tan a} = \frac{2 \tan b}{1 - \tan^2 b}, \text{ or } \frac{\frac{130.08 - E}{N + 36.95} - \frac{E + 106.27}{N + 127.04}}{1 + \left(\frac{130.08 - E}{N + 36.95}\right)\left(\frac{E + 106.27}{N + 127.04}\right)} = \frac{-2E}{1 - \left(\frac{E}{N}\right)^2}$$

$$\text{which reduces to } 23.81N^3 + 12,598.6867N^2 - 12,598.6867E^2 + 37035.4592EN + 163.99EN^2 + 23.81NE^2 + 163.99E^3 = 0, \text{ or}$$

$$E\sqrt{95.264 - E^2} + 61.2451\sqrt{95264 - E^2} + 421.822E + 32406.9 - 0.680358E^2 = 0$$

(N can be eliminated by substituting $\sqrt{308.649^2 - E^2}$ for N)

Some readers have recommended using the "Goal Seek" feature of Excel. This problem lends itself ideally to that solution. Let your first column be E, define N in the next column and set up the formulas for all the other terms in successive columns. Let "goal seek" find the value for E that makes the above long formula equal to zero. E is solved as -66.2084935 and N is 301.4641614 by "Goal Seek."

N=301.46 and E= -66.21 are close enough. PQ & PR are 587.98.

Alternatively, you can type the long equation into www.wolframalpha.com.