Solution
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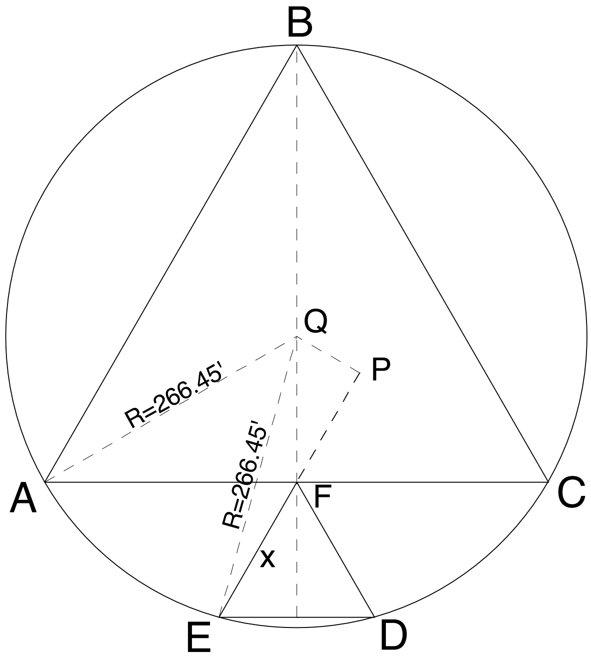

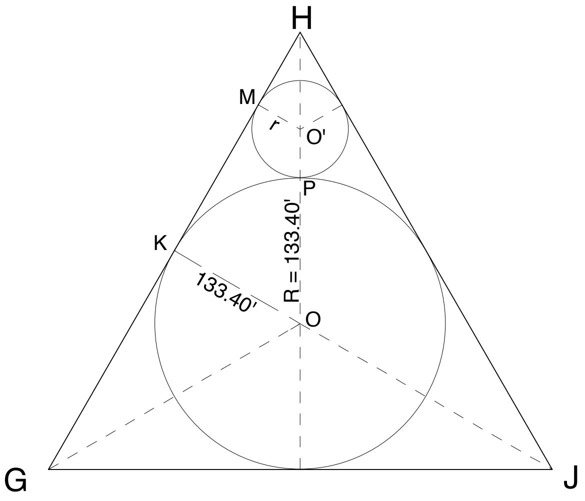

AQ = 266.45

Because AQF is a 30°–60°–90° triangle with sides in the ratio 1:2:$\sqrt{3}$,

QF = ½ AQ = 133.225

Extending EF to P so that angle QPF is a right angle,

triangle FQP is also a 30°–60°–90° triangle

QP = ½ QF = 66.6125

and FP = $(\sqrt{3}/2)$QF = 115.3762

In triangle EQP, $EQ^2 = QP^2 + EP^2$ where EP = x + 115.3762

$266.45^2 = 66.6125^2 + (x + 115.3762)^2$

yielding $x^2 + 230.7524x + 115.3762^2 + 66.6125^2 - 266.45^2 = 0$

$x^2 + 230.7524x - 53{,}246.7098 = 0$, a quadratic equation in x

$$x = \frac{-230.7524 \pm \sqrt{230.7524^2 + (4)(53246.7098)}}{2}$$, and x= 142.613

Within triangle GHJ, GOK and HO'M are 30°–60°–90° triangles

OK= R =133.40, GO=2R= HO=266.80, making HP = 133.40

HO' = 133.40 – r, with triangle HO'M being a 30°–60°–90° triangle

MO' = ½ (133.40 – r) and HM = $(\sqrt{3}/2)(133.40 - r)$

$$\frac{r}{133.40} = \frac{133.40 - r}{266.80}$$

$266.80\,r = 133.40^2 - 133.40\,r$

400.20 r = 17795.56

r = 44.467 (which is exactly 1/3 R)

A side of the equilateral triangle GHJ is $(2)(\sqrt{3})(133.40) = 462.111$