Solution 172

by Dave Lindell, L.S.

The radius of any inscribed circle, also known as the incircle, is given by

$$r = \sqrt{\frac{(s-a)(s-b)(s-ac)}{s}}$$, where s is the semiperimeter: $s = \frac{a+b+c}{2}$

We have b, c and r in this problem.

$s = \frac{1}{2}(a + 528.390 + 622.118) = 575.254 + \frac{1}{2}a$

$r^2 (575.254+\frac{1}{2}a)=(575.254-\frac{1}{2}a)(575.254+\frac{1}{2}a-528.39)(575.254+\frac{1}{2}a-622.118)$

$147.288^2(575.254)+10{,}846.8775\ a = (575.254-\frac{1}{2}a)(46.864+\frac{1}{2}a)(\frac{1}{2}a-46.864)$

which reduces to: $a^3 - 1150.508\ a^2 + 77990.082\ a + 109{,}946{,}496 = 0$

Any cubic equation of the form $ax^3 + bx^2 + cx + d = 0$ can be reduced to the form $y^3 + 3py + q = 0$ by letting $y = x + p/3a$

so that $p = (1/9a^2)(3ac - b^2)$ and $q = 1/27(2\ b^3 - 9abc + 27a^2d)$

(This solution does not require the coefficient of a^3 to be unity.)

$$p = \frac{(3)(1)(77{,}990.082) - (-1.150.508)^2}{(9)(1)} = -121{,}077.6013$$

$$q = \frac{(2)(-1{,}150.508)^3 - (9)(-1{,}150.508)(77{,}990.082) + 27(109{,}942{,}496)}{27}$$

$=27{,}045{,}131.53$

Letting $r = \sqrt{(-p^3)}$ = 42,130,412.63 and $\Phi = \cos^{-1}(-q/2r)$ = 108°43'17.578"

the solutions to the cubic equation in y are $y = 2\sqrt[3]{r}\cos(\frac{\Phi}{3})$,

$y = 2\sqrt[3]{r}\cos(\frac{\Phi+360^\circ}{3})$ and $y = 2\sqrt[3]{r}\cos(\frac{\Phi+720^\circ}{3})$

y = 561.2924, –636.941 and 75.6486,

but, $y = x + p/3a$, or $x = y + 383.5027$

x = 944.795, –253.438 and 459.1513, the latter being the solution for this problem

Solution 173

by Benjamin Bloch, Ph.D.

a) **1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9**; *1, 1, 2, 3, 5, 8, ...*

The SDQ period is 24 terms. Thus, nature has a built-in growth cycle of 24 terms.

b) **1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9**; *1, 1, 2, 3, 5, 8, ...*

I have shown the first half of the twenty-four-term pattern in blue and the second half in red. The first and thirteenth terms add to SDQ 9, the second and fourteenth term add to SDQ 9, and so forth. This can be easily seen by writing the second half of the series directly below the first half, thus:

1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9,

8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9;