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by Dave Lindell, L.S.

The radius of any inscribed circle, also known as the incircle, is given by

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}, \text{ where } s \text{ is the semiperimeter: } s = \frac{a+b+c}{2}$$

We have b, c and r in this problem.

$$s = \frac{1}{2}(a + 528.390 + 622.118) = 575.254 + \frac{1}{2}a$$

$$r^2 (575.254 + \frac{1}{2}a) = (575.254 - \frac{1}{2}a)(575.254 + \frac{1}{2}a - 528.39)(575.254 + \frac{1}{2}a - 622.118)$$

$$147.288^2(575.254) + 10,846.8775 a = (575.254 - \frac{1}{2}a)(46.864 + \frac{1}{2}a)(\frac{1}{2}a - 46.864)$$

$$\text{which reduces to: } a^3 - 1150.508 a^2 + 77990.082 a + 109,946,496 = 0$$

Any cubic equation of the form $ax^3 + bx^2 + cx + d = 0$ can be reduced to the

form $y^3 + 3py + q = 0$ by letting $y = x + p/3a$

so that $p = [1/(9a^2)](3ac - b^2)$ and $q = (1/27)(2b^3 - 9abc + 27a^2d)$

(This solution does not require the coefficient of a^3 to be unity.)

$$p = \frac{(3)(1)(77,990.082) - (-1,150.508)^2}{(9)(1)} = -121,077.6013$$

$$q = \frac{(2)(-1,150.508)^3 - (9)(-1,150.508)(77,990.082) + 27(109,942,496)}{27}$$

$$= 27,045,131.53$$

Letting $r = \sqrt{(-p^3)} = 42,130,412.63$ and $\Phi = \cos^{-1}(-q/2r) = 108^\circ 43' 17.578''$

the solutions to the cubic equation in y are $y = 2\sqrt[3]{r} \cos\left(\frac{\Phi}{3}\right),$

$$y = 2\sqrt[3]{r} \cos\left(\frac{\Phi + 360^\circ}{3}\right) \text{ and } y = 2\sqrt[3]{r} \cos\left(\frac{\Phi + 720^\circ}{3}\right)$$

$$y = 561.2924, -636.941 \text{ and } 75.6486,$$

but, $y = x + p/3a$, or $x = y + 383.5027$

$x = 944.795, -253.438$ and 459.1513, the latter being the solution for this problem