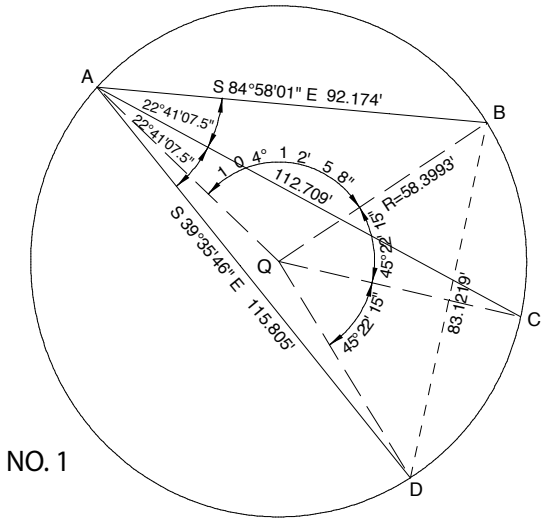
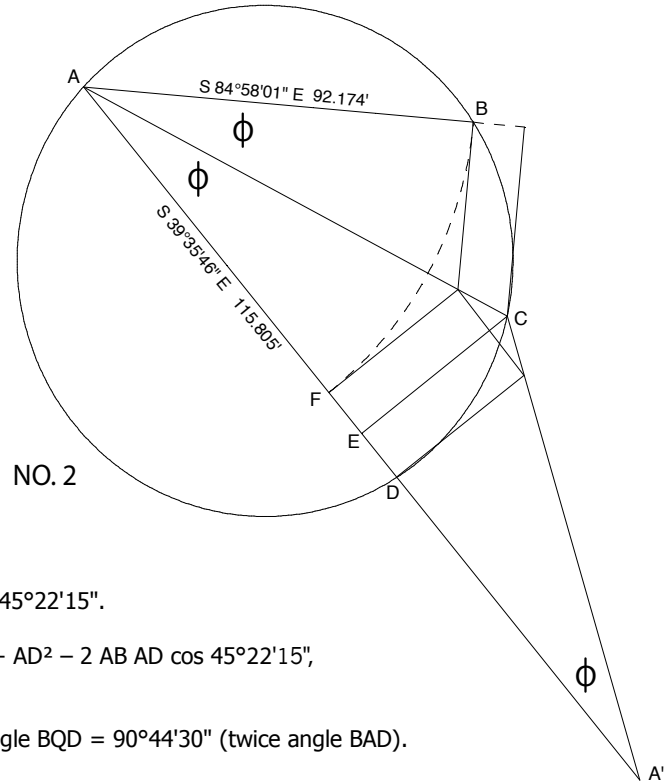


# Solution 168

by Dave Lindell, L.S.



NO. 1



NO. 2

By difference of bearings, angle  $BAD = 45^{\circ}22'15''$ .

Using the Law of Cosines,  $BD^2 = AB^2 + AD^2 - 2 AB AD \cos 45^{\circ}22'15''$ ,  
so  $BD = 83.1219$

In triangle  $BQD$ ,  $BQ = QD = \text{radius}$ . Angle  $BQD = 90^{\circ}44'30''$  (twice angle  $BAD$ ).

Chord  $BD = 2 R \sin 45^{\circ}22'15''$ , so that  $R = 58.3993'$

In triangle  $AQB$  by the Law of Cosines,  $92.174^2 = R^2 + R^2 - 2 R^2 \cos \text{angle } AQB$ ,  
so that angle  $AQB = 104^{\circ}12'58''$ .

Angle  $AQB + \text{angle } BQC = 104^{\circ}12'58'' + 45^{\circ}22'15'' = 149^{\circ}35'13''$

Chord  $AC = 2(58.3993) \sin[\frac{1}{2}(149^{\circ}35'13'')] = 112.709'$

(Note: This is a weak solution, using cosines of angles between  $120^{\circ}$  &  $180^{\circ}$ .)

Alternatively,

Extend line  $AD$  to intersect  $CA'$ , where  $CA' = AC$ . Construct  $AF = AB$ .

$AA' = 2 AC \cos \phi = AD + DA'$ , so that  $2AC \cos \phi - AD = DA'$

$AE - \frac{1}{2} FD = AF$ , and  $FD = AD - AF$

$2 AE - FD = 2 AF$ , and  $2AE - (AD - AF) = 2 AF$ , and  $2 AE - AD = AF$

but  $AE = 2 AC \cos \phi$ , so  $DA' = AF$

Therefore,  $2 AC \cos \phi = AD + AF$ , but  $AF = AB$

so,  $2 AC \cos \phi = AD + AB$ , or  $AC = (AD + AB) / 2 \cos \phi$

(This is known as the Three Chord Lemma)



**Solution**  
**169**

by Benjamin Bloch, Ph.D.

**Solution to Problem 169**

a) Here  $x = 2$ , and  $n = 15$ .  $(15-2)/6 = 2$  R 1, so that  $SDQ(2) = 2$ , and  $R = 1$ . From the table the answer is **8**. Thus,  $SDQ(2^{15}) = 8$ , and since  $SDQ(32,868) = 27 \Rightarrow 9$  this answer must be *incorrect*.

**Correct answer:**  $(32,768) \Rightarrow 26 \Rightarrow 8$ .

b)  $16^7 = ?$  268,335,456.

$SDQ(16) = 7$ . For  $n = 7$  we need not use the remainder calculation. From the table the answer is **7**.

$SDQ(268,335,456) = 42 \Rightarrow 6$ . So this answer is *incorrect*.

**Correct answer:**  $(268,435,456) \Rightarrow 43 \Rightarrow 7$ .

c)  $8^{23} = ?$  590,295,810,358,305,651,712  $\Rightarrow 85 \Rightarrow 13 \Rightarrow 4$ .

$(23 - 2)/6 = 3$  R = 3

From the table we get **8**. So this answer is *incorrect*.

**Correct answer:**  $(590,295,810,358,705,651,712) \Rightarrow 89 \Rightarrow 17 \Rightarrow 8$ .

d)  $147^{28} = ?$  4,840,445,926,998,527,143,180,132,566,802,461,408,607,116,960,093,883,732,904,561  $\Rightarrow 263 \Rightarrow 11 \Rightarrow 2$

$SDQ(147) = 12 \Rightarrow 3$

$(28-2)/6 = 4$  R 2

From the table the answer is **9**. Therefore the answer given is *incorrect*.

**Correct answer:**  $(4,840,445,926,998,527,143,180,132,566,802,461,408,607,116,960,091,883,732,904,561) \Rightarrow 261 \Rightarrow 9$ .