

In order to establish a single centerline it will be necessary to change the offsets to the found points as if you were to measure 33.00 feet from each one towards the centerline. A least squares adjustment will give the most probable best fit line to the data. The new values are:

1+01.54, 7.86; 1+54.06, 5.81; 2+06.41, 4.06; 2+43.48, 2.45; 2+91.03, 0.53; 3+38.47, -1.43; 4+01.12, -3.41; 4+53.53, -5.86; 5+10.93, -8.03

Letting the line F-G be North (it can be rotated to any bearing), the station be "y" and the offset be "x",  $\overline{x} = 0.22$ ,  $\overline{y} = 300.0633$  (the mean of each set). The slope, m, and y-intercept, b, of the line are calculated as

$$m = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}, \quad b = \overline{y} - m\overline{x}$$

$$m = \frac{-5330.2955 - \frac{(1.98)(2700.57)}{9}}{230.7962 - \frac{(1.98)^2}{9}} = -25.71803034 = \tan - 87^{\circ}46'23.8''$$

(the angle is measured from "east", or 0° as in trigonometry)

$$b = \overline{y} - m\overline{x}$$
,  $b = 300.0633 + (25.71803034)(0.22) = 305.7213$ 

The centerline bears N 2°13'36.2" W from a point 11.887 East of point F.

The line has slope-intercept form y = -25.71803034x + 305.7213

To convert to the general form Ax + By + C = 0, the slope is  $-\frac{A}{B} = -25.71803034$  (the tangent of the *azimuth* is  $-\frac{B}{A}$ ) the x-intercept is  $-\frac{C}{A} = 11.8874$ , and the y-intercept is  $-\frac{C}{B} = 305.7213$ 

A= 25.71803034 B

Continued  $\gg$ 





 $-C = 305.7213B, \quad B = \frac{305.7213}{305.7213} = 1$ 

 $-\frac{A}{B} = -25.71803034, \quad A = 25.71803034$ 

 $\sqrt{A^2 + B^2} = 25.737465$ 

The distance from any point  $p_i$  to the line is given by

 $\frac{\mathcal{A}x_i + \mathcal{B}y_i + \mathcal{C}}{\sqrt{\mathcal{A}^2 + \mathcal{B}^2}}$  , where  $x_i$  and  $y_i$  are the original coordinates of the point

For the first point, for example,  $\frac{(25.718)(40.86) + 101.54 - 305.7213}{25.737465} = 32.896$ 

 $\frac{(25.718)(-27.19)+154.06-305.7213}{25.737465} = -33.062$ 

 $\frac{(25.718)(37.06) + 206.41 - 305.7213}{25.737465} = 33.173$ 

 $\frac{(25.718)(-30.55) + 243.48 - 305.7213}{25.737465} = -32.945$ 

 $\frac{(25.718)(33.53) + 291.03 - 305.7213}{25.737465} = 32.934$ 

 $\frac{(25.718)(-34.43) + 338.47 - 305.7213}{25.737465} = -33.132$ 

 $\frac{(25.718)(29.59) + 401.12 - 305.7213}{25.737465} = 33.274$ 

 $\frac{(25.718)(-38.86)+453.53-305.7213}{25.737465}=-33.088$ 

 $\frac{(25.718)(24.97) + 510.93 - 305.7213}{25.737465} = 32.924$ 



- a) The first 30 terms of the Fibonacci Series are: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1,597, 2,584, 4,181, 6,765, 10,946, 17,711, 28,657, 46,368, 75,025, 121,393, 196,418, 317,811, 514,229, 832,040, ...
- b) 46,368/28,657 = 1.6180+ This is the Golden Proportion, F.
- c) 28,657/46,368 = 0.6180+ This is the inverse of the Golden Proportion, F-1.
- d) A piano keyboard had 5 black keys, 8 white keys, and 13 black and white keys that together make up the octave, following the Fibonacci sequence rule. As we move out on the sequence, the ratio of their terms approaches the Golden Proportion, 1.618, while their inverse ratio approaches 0.618. Thus, even using the sixth and seventh terms, 8 and 13, we get that 13/8 = 1.625 and 8/13 = 0.615. 13 divided by 8 differs from F, the Golden Proportion by less than one half of one percent.

Note: It is possible to start the Fibonacci series either with 0 and 1, or with 1 and 1, or with 1 and 2, each yielding the same resulting series.

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