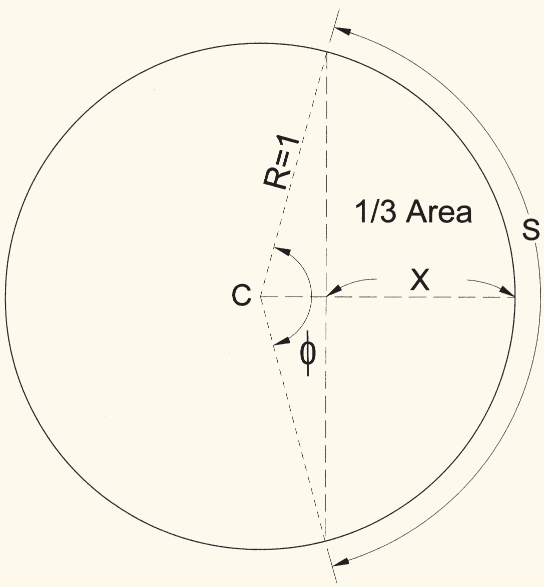
Solution to Problem 130



Let $R = 1$ mile

The area of a whole circle is $\pi R^2 = (2\pi R)(½ R) = ½CR$, where C is the circumference of the circle.

The area of a sector is to the area of the entire circle as the arc of the sector is to the total circumference of the circle.

The arc of the sector is $S = R\phi$, where ϕ is in radians.

The area of the sector is $½ S R = ½ R^2\phi$.

The area of the triangle under the segment is $½ R^2 \sin \phi$.

The area of ½ the shaded lune is the area of the sector minus the area of the triangle, $½ R^2\phi - ½ R^2\sin \phi = ½R^2(\phi - \sin \phi)$

We want to know for what value of ϕ the area is equal to $1/3 \pi R^2$, so that $½R^2(\phi - \sin \phi) = 1/3\pi R^2$, which reduces to $\phi - \sin \phi - 2/3\pi = 0$

This is not directly solvable, but by "trial and error", or iteration,

For $\phi = 2$, the equation yields –1.003692529

For $\phi = 3$, the equations yields 0.76448489

By interpolation, ϕ of 2.567642431 yields –0.069706197

Further interpolation finally gives ϕ =2.605325696 or ϕ = 149°16′27″,

from which R–X is 5280 cos 74°38′13.5″ or 1398.841′

The centers are then 2797.683′ apart.

Solution to Problem 131

You can immediately place any number in its single digit column. When you examine the **1** column you'll notice that each number in that column, 10, 19, 28, and 37, adds to **1** when you add their digits until you get a single digit. For example 10's digits 1 and 0 add to **1**. The number 19's digits 1 and 9 add to 10, and those digits 1 and 0 add to **1**. The number 28's digits add to 10, and those digits add to **1**. The final single digit we call the SDQ or single digit quality. We write this mathematically as 10=>**1**, 19=>10=>**1**, 28=>10=>**1**, and 37=>10=>**1**.

86=>14=>**5**, and thus belongs in the single digit **5** column
175=>13=>**4**, and thus belongs in the single digit **4** column
4,688=>26=>**8**
53,493=>24=>**6**
106,441=>16=>**7**
7,121,368=>28=>10=>**1**
22,719,854=>38=>11=>**2**
958,877,535=>57=>12=>**3**
1,476,856,872=>54=>**9**