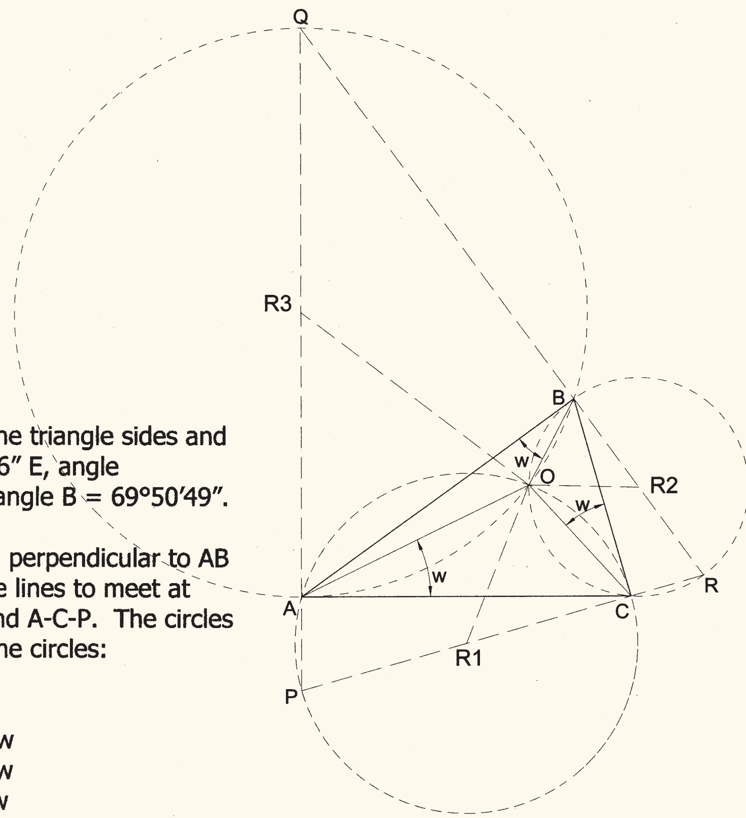
PROBLEM CORNER

Solution to Problem 129



From the given coordinates calculate bearings for the triangle sides and interior angles for the triangle: AC= East; AB = N 53°54′06″ E, angle A=36°05′54″; CB = N 15°56′43″W, angle C = 74°03′17″; angle B = 69°50′49″.

Construct a line perpendicular to line AC through A, perpendicular to AB through B, and perpendicular to BC through C. Extend the lines to meet at points P, Q, and R. Draw a circle through A-B-Q, B-C-R and A-C-P. The circles intersect at the desired point, "O". Calculate the radii of the circles: R1=3016.044, R2 = 1938.725 and R3 = 5041.423.

AO is a chord of circle R3: AO = (2)(5041.423)sin w
OB is a chord of circle R2: OB = (2)(1938.725)sin w
OC is a chord of circle R1: OC = (2)(3016.044)sin w

$$\frac{OC}{6032.088} = \frac{OB}{3877.550} = \frac{AO}{10082.846}$$

OC = 1.555644157(OB), AO = 2.6003136(OB)

By the Law of Cosines: $(AC)^2 = (AO)^2 + (OC)^2 - (2)(AO)(OC)\cos(A\text{-}O\text{-}C)$
Angle A-O-C = 180° – w – (74°03′17″ – w) = 105°56′43″

$$5800^2 = [\,2.6003136^2 + 1.555644157^2 + (2)(1.55544157)(2.60031376)(0.274719146)]\,(OB)^2$$

$$5800^2 = 11.40422683\,(OB)^2$$

OB = 1717.493, so that OC = 2671.808 and AO = 4466.021

$$\cos w = \frac{(AO)^2 + (AC)^2 - (OC)^2}{(2)(AO)(AC)} = \frac{4466.021^2 + 5800^2 - 2671.808^2}{(2)(4466.021)(5800)}$$

w = 26°17′28″

(Note: cot w = cot A + cot B + cot C; how's that for simplicity?

The angles shown are counterclockwise angles within the triangle. There is also a set of clockwise angles with the same value which intersect in a different point. The intersection points are called Brocard points.)