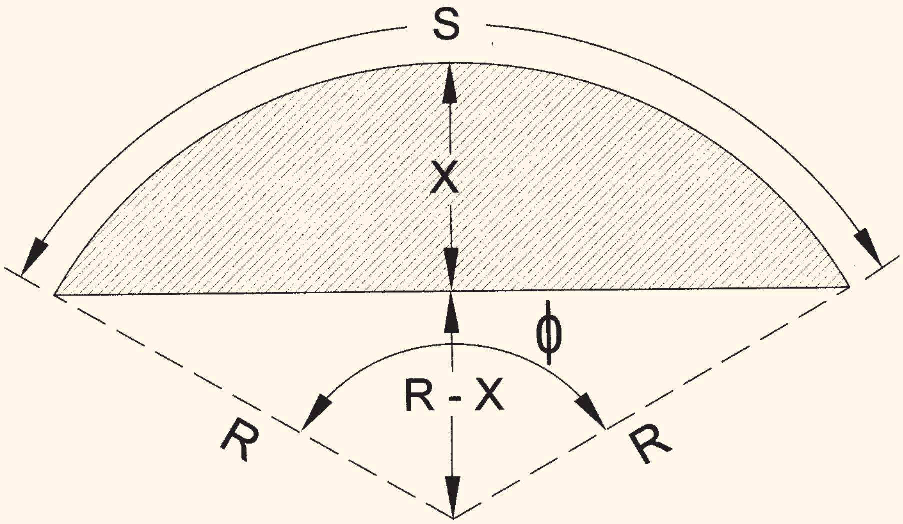
PROBLEMCORNER

Solution to Problem 123



The area of the whole circle is $\pi R^2 = (2\pi R)(\frac{1}{2} R) = \frac{1}{2}CR$, where C is the circumference of the circle.

The area of the sector is to the area of the entire circle as the arc of the sector is to the total circumference of the circle.

The arc of the sector is $S = R\phi$, where ϕ is in radians.

The area of the sector is $\frac{1}{2} S R = \frac{1}{2} R^2\phi$.

The area of the triangle under the segment is $\frac{1}{2} R^2 \sin \phi$.

The area of the shaded segment is the area of the sector minus the area of the triangle, $\frac{1}{2} R^2\phi - \frac{1}{2} R^2\sin \phi = \frac{1}{2}R^2(\phi - \sin \phi)$

We want to know for what ϕ the area is equal to $\frac{1}{4}\pi R^2$,

so that $\frac{1}{2}R^2(\phi - \sin \phi) = \frac{1}{4}\pi R^2$, which reduces to $\phi - \sin \phi - \frac{1}{2}\pi = 0$

This is not directly solvable. A quick plot of ϕ values for 0, $\frac{1}{2}\pi$ and π shows the value to be near $\frac{3}{4}\pi$.

For $\frac{3}{4}\pi$: 2.35619449 – 0.707106781 – 1.570796327 = 0.078291382

For $\frac{2}{3}\pi$: 2.094395102 – 0.866025404 – 1.570796327 = – 0.342426629

Linear interpolation gives ϕ = 2.307476262 which yields

2.307476262 – 0.740703164 – 1.570796327 = – 0.004023229

Another linear interpolation with $\frac{3}{4}\pi$ gives ϕ = 2.309857427 yielding

2.309857427 – 0.739101322 – 1.570796327 = – 0.000040222

Using the two values closest to zero yields ϕ = 2.309881484 for

2.309881484 – 0.739085117– 1.570796327 = 0.00000004

(This value is within 0.01″ of ϕ which is 132°20′47.25″, close enough!)

$R - X = R \cos \frac{1}{2} \phi$ = 0.403972742 R so that X = 0.596027258 R

Solution to Problem 124

SOLUTION TO PROBLEM NO. 124

The area of the inner cross-hatched circle is πr^2, where $r = 3$, or $\pi 3^2$

The area of the outer cross-hatched ring is

$$\pi 5^2 - \pi 4^2 = 25\pi - 16\pi = 9\pi = 3^2\pi$$

Their areas are equal.