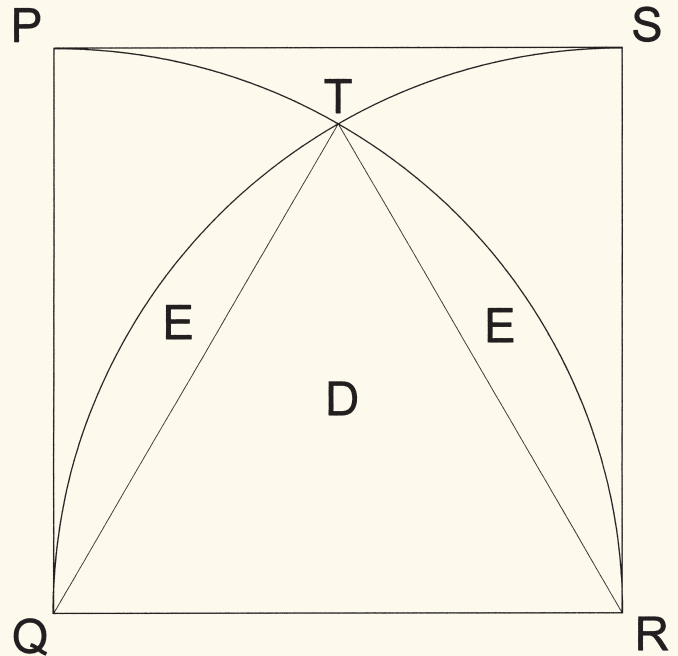
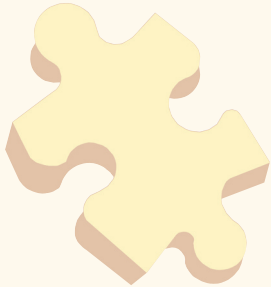




Solution to Problem 113



Let square PQRS be *1 mile* on a side.

(1) Area A + 4 x area B + 4 x area C = 1 square mile.

(2) Area A + 3 x area B + 2 x area C = $\frac{1}{4}$ of a circle = $\frac{\pi}{4}$

(3) Area A + 2 x area B + area C = 2 x area E + area D

Triangle QTR is equilateral and has an area of $\frac{\sqrt{3}}{4}$

2 area E + area D = Sector TQR + sector QRT – triangle QTR

Sector TQR = Sector QRT = $\frac{\pi}{6}$

(1) $A + 4B + 4C = 1$

(2) $A + 3B + 2C = \frac{\pi}{4}$

(3) $A + 2b + C = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

Combining (1) and (2), $B + 2C = 1 - \frac{\pi}{4}$ (4)

Combining (2) and (3), $B + C = \frac{\pi}{4} - \frac{\pi}{3} + \frac{\sqrt{3}}{4}$ (5)

Combining (4) and (5), $C = 1 - \frac{\sqrt{3}}{4} - \frac{\pi}{6} = 0.043388523$ sq. mi.

By substitution of C into (5), $B + 1 - \frac{\sqrt{3}}{4} - \frac{\pi}{6} = \frac{\pi}{4} - \frac{\pi}{3} + \frac{\sqrt{3}}{4}$

and $B = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 = 0.127824792$ sq. mi.

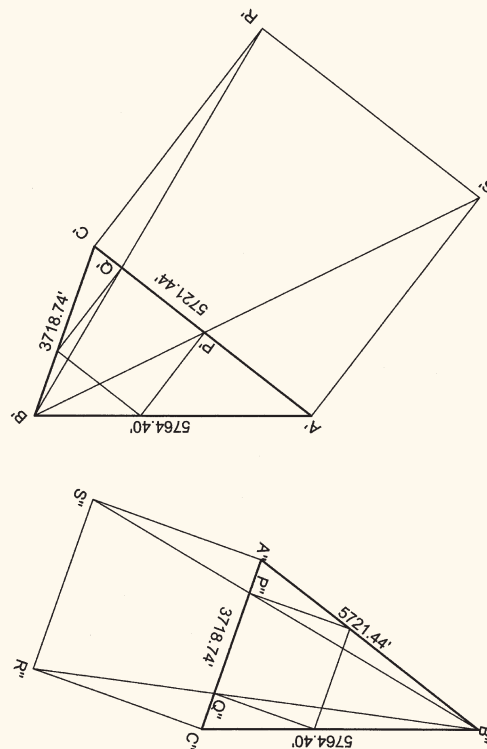
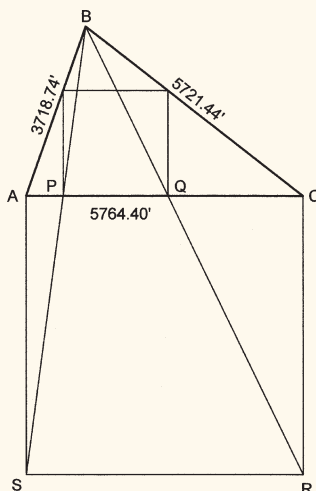
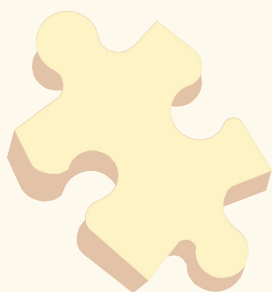
By substitution of B and C into (2),

$A + 3(\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1) + 2(1 - \frac{\sqrt{3}}{4} - \frac{\pi}{6})$

And $A = 1 + \frac{\pi}{3} - \sqrt{3} = 0.315146744$ sq. mi.



Solution to Problem 114



First, solve for the angles in the triangle by the Law of Cosines:

$$A = 70^{\circ}29'02.4'', \quad B = 71^{\circ}44'11.5'', \quad C = 37^{\circ}46'46.1''$$

Construct square A-C-R-S on side AC. Construct lines BS and BR, intersecting AC at P and Q. Construct perpendiculars to AC at P and Q. The intersections with sides AB and BC are the corners of the square as are points P and Q.

Let AC bear East. Traverse from A to B (or C to B, or both for a check) using the complements of the angles calculated for the triangle as bearings. Traverse to S and R, or just assign coordinates to them. Inverse from S to B and from R to B.

$SB = N 7^{\circ}38'00'' E$, $RB = N 26^{\circ}00'19'' W$ (No distances are needed)

$$AP = 5764.40 \times \tan 7^{\circ}38'00'' = 772.56'$$

$$CQ = 5764.40 \times \tan 26^{\circ}00'19'' = 2812.14'$$

$$PQ = 5764.40 - 772.56 - 2812.14 = 2179.70'$$

Now, do the same for the other two sides: (Just substitute A' or A'' for A, etc.)

The bearing for $S'B'$ is $N 26^{\circ}12'55'' E$ and for $R'B'$ is $N 7^{\circ}10'43'' W$
 $A'P' = 2817.195'$, $C'Q' = 720.62'$, making $P'Q' = 2183.625'$

The bearing for $S''B''$ is $N 11^{\circ}05'05'' E$ and for $R''B''$ is $N 11^{\circ}52'57'' W$
 $A''P'' = 728.565'$, $C''Q'' = 782.48'$, making $P''Q'' = 2207.695'$

The square constructed on the shortest side is the largest.