

## Solution to Problem 109

The formula  $A = P ( 1 + r/n )^{nt}$ , where A is the amount, P is the principal, r is the rate, n is the number of compounding periods and t, the time period, is used to calculate compound interest.

In your case,  $P = 10,000,000$ ,  $r = 100\% = 1.00$ , n is the number you choose and  $t = 1$ , one year.

Compounded monthly,  $A = \$10,000,000(1+1/12)^{12} = \$26,130,352.90$

Weekly,  $A = 10,000,000(1+1/52)^{52} = 26,925,969.54$

Daily,  $A = 10,000,000(1 + 1/365)^{365} = 27,145,674.82$

Hourly,  $A = 10,000,000(1 + 1/8760)^{8760} = 27,181,266.92$

By the minute,  $A = 10,000,000(1 + 1/525600)^{525600} = 27,182,792.43$

By the second,  $A = 10,000,000(1 + 1/31536000)^{31536000} = 27,182,817.85$

For 100,000,000,  $A = 10,000,000(1 + 1/100,000,000)^{100,000,000} = 27,182,818.15$

Recognize this number?

It is approaching  $10,000,000e$ , where e is the base of the natural logarithms.

In fact, the limit of  $(1 + w)^{1/w}$  as w approaches zero is the definition of e.

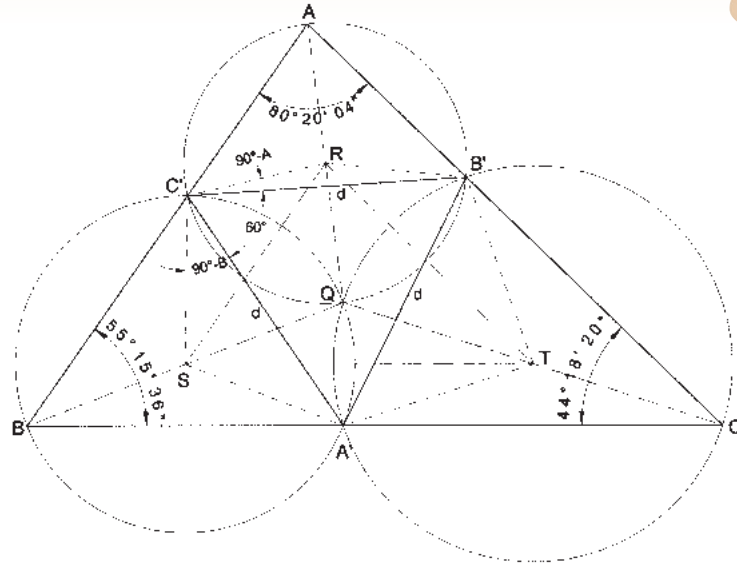
(e = 2.71828 18284 59045 23536 02874 71352 66249 77572 ...)

(The real answer to this problem is "zero". It was a dream, remember?)





## Solution to Problem 110



Construct circles with centers at R, S and T that circumscribe triangles A-B'-C', B-C'-A' and C-A'-B' respectively. Because angle A-C'-Q is a right angle, line A-Q must be a diameter, as must B-Q and C-Q. Draw lines R-S, S-T and T-R. Line R-S is perpendicular to a line joining C'-Q, a chord of the circle with center at S or R, and therefore parallel with line A-B'. Side R-S can be shown by similar triangles to be one-half the length of side A-B, S-T half that of B-C and T-R half that of A-C.

From the given sides, calculate the angles of triangle A-B-C by the Law of Cosines:  $A = 80^{\circ}20'04''$ ,  $B = 55^{\circ}15'36''$ ,  $C = 44^{\circ}18'20''$ .

Angle C'-R-B' = twice angle A, and angle C'-S-A' = twice angle B

$(C'-B') = 2 (C'-R) \sin A$ , so that  $(C'-R) = (C'-B')/2 \sin A = d/2 \sin A$

$(C'-A') = 2 (C'-S) \sin B$ , so  $(C'-S) = (C'-A')/2 \sin B = d/2 \sin B$

where d is the side of the equilateral triangle

Angle R-C'-B' = angle R-B'-C' =  $90^{\circ} - \text{angle A}$

Angle S-C'-A' = angle S-A'-C' =  $90^{\circ} - \text{angle B}$

Angle R-C'-S =  $90^{\circ} - \text{angle A} + 90^{\circ} - \text{angle B} + 60^{\circ}$

=  $180^{\circ} - \text{angle A} - \text{angle B} + 60^{\circ}$

but  $180^{\circ} - \text{angle A} - \text{angle B} = \text{angle C}$

so angle R-C'-S = angle C +  $60^{\circ}$

From the Law of Cosines,  $(R-S)^2 = (C'-S)^2 + (C'-R)^2 - 2(C'-S)(C'-R) \cos(C+60^{\circ})$

$425^2 = d^2/4 \sin^2 B + d^2/4 \sin^2 A - (2)(d/2 \sin B)(d/2 \sin A) \cos(C+60^{\circ})$

Multiplying both sides by  $4 \sin^2 A \sin^2 B$ ,

$425^2(4 \sin^2 A \sin^2 B) = d^2 \sin^2 A + d^2 \sin^2 B - 2d^2 \sin B \sin A \cos(C+60^{\circ})$

$d^2 = 425^2(4)(\sin^2 A \sin^2 B) / [\sin^2 A + \sin^2 B - 2 \sin B \sin A \cos(C+60^{\circ})]$

$d = 850 \sin A \sin B / \sqrt{[\sin^2 A + \sin^2 B - 2 \sin B \sin A \cos(C + 60^{\circ})]}$

with all values on the right side known,  $d = 481.283'$