



Solution to Problem 67

The most probable value is given by a least squares solution.

Let $L_1 = AB$, $L_2 = BC$, $L_3 = CD$ with residuals r_1 , r_2 and r_3 , respectively.

Set up condition equations:

$$AB + r_1 = L_1, \text{ so } r_1 = L_1 - AB = L_1 - 49.897$$

$$BC + r_2 = L_2, \text{ so } r_2 = L_2 - BC = L_2 - 99.846$$

$$CD + r_3 = L_3, \text{ so } r_3 = L_3 - CD = L_3 - 99.931$$

$$AC + r_1 + r_2 = L_1 + L_2, \text{ so } r_1 + r_2 = L_1 + L_2 - AC = L_1 + L_2 - 149.797$$

$$BD + r_2 + r_3 = L_2 + L_3, \text{ so } r_2 + r_3 = L_2 + L_3 - BD = L_2 + L_3 - 199.812$$

To get a least squares solution the sum of the squares of the residuals must be minimized, to wit:

$$\phi = r_1^2 + r_2^2 + r_3^2 + (r_1 + r_2)^2 + (r_2 + r_3)^2 = (L_1 - 49.897)^2 + (L_2 - 99.846)^2 + (L_3 - 99.931)^2 + (L_1 + L_2 - 149.797)^2 + (L_2 + L_3 - 199.812)^2 \text{ must be minimized}$$

The partial derivative evaluated with respect to each estimate and equated to zero will minimize ϕ :

$$\frac{\partial \phi}{\partial L_1} = 2(L_1 - 49.897) + 2(L_1 + L_2 - 149.797) = 0$$

$$\frac{\partial \phi}{\partial L_2} = 2(L_2 - 99.846) + 2(L_1 + L_2 - 149.797) = 0$$

$$\frac{\partial \phi}{\partial L_3} = 2(L_3 - 99.931) + 2(L_2 + L_3 - 199.812) = 0$$

$$L_1 - 49.897 + L_1 + L_2 - 149.797 = 0, \text{ or } 2L_1 + L_2 = 199.694$$

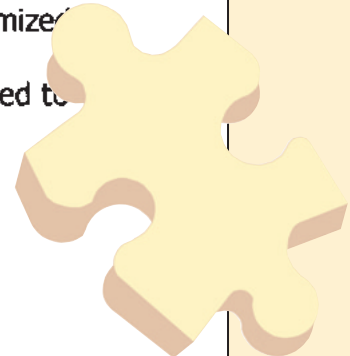
$$L_2 - 99.846 + L_1 + L_2 - 149.797 + L_2 + L_3 - 199.812 = 0, \text{ or } L_1 + 3L_2 + L_3 = 499.455$$

$$L_3 - 99.931 + L_2 + L_3 - 199.812 = 0, \text{ or } L_2 + 2L_3 = 299.743$$

$$2L_1 + 6L_2 + 2L_3 - 2L_1 - L_2 = 699.216 = 5L_2 + 2L_3$$

$$\text{but } 5L_2 + 10L_3 = 1498.715, \text{ so } 8L_3 = 799.499 \text{ and } L_3 = 99.9374$$

$$\text{From which } L_1 = 49.9129, L_2 = 99.8683 \text{ and } A-D = 249.7185$$

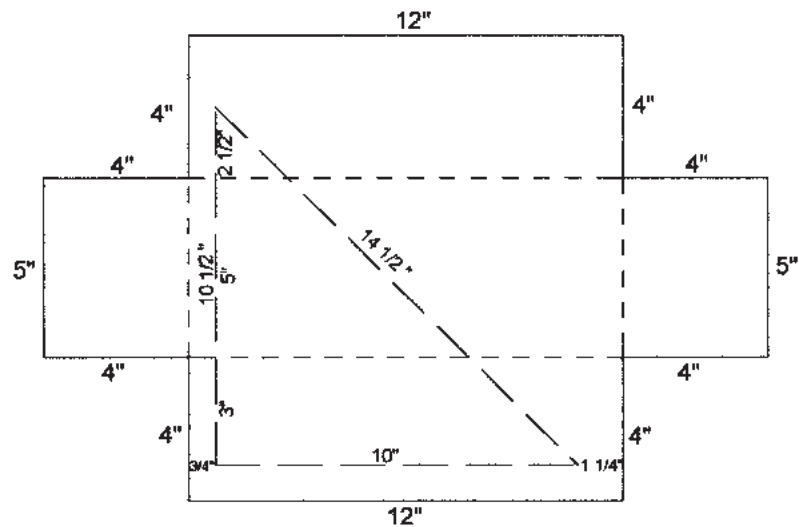




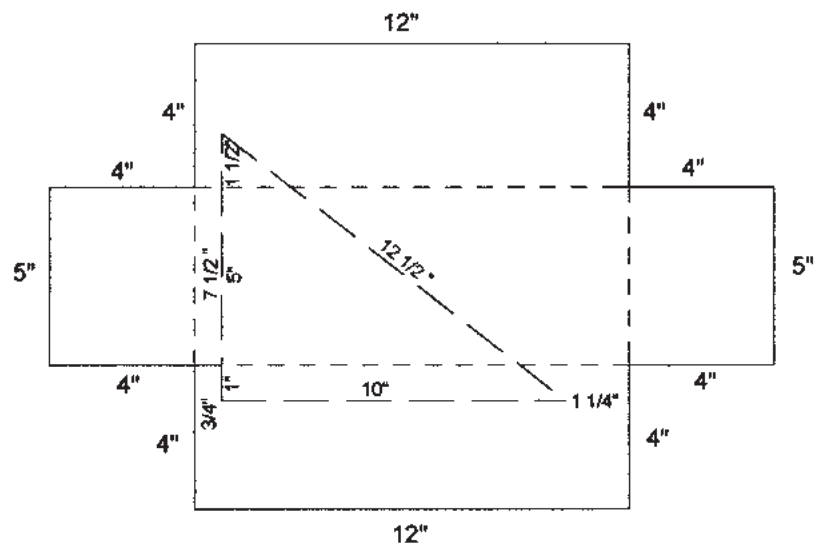
Solution to Problem 68

"OPEN" THE BOX SO IT LAYS FLAT. THERE ARE TWO POSSIBILITIES.

DOWN THE WALLS AND ACROSS THE FLOOR:



UP THE WALLS AND ACROSS THE CEILING:



THE CEILING ROUTE IS SHORTER