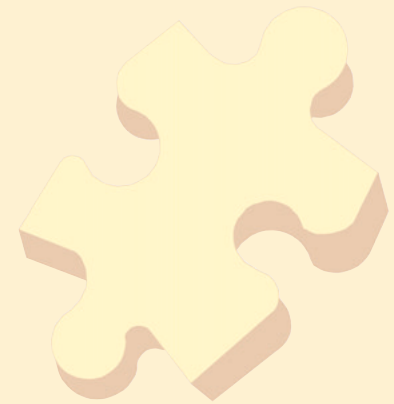
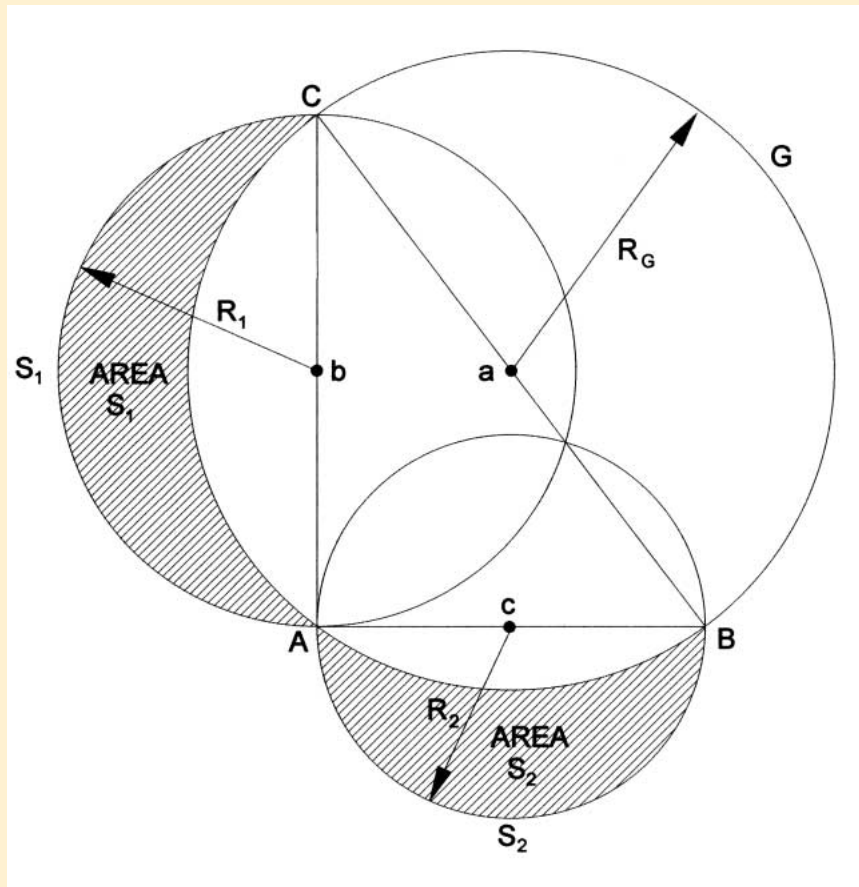




# Solution to Problem 53



Area  $S_1$  + area  $S_2$  = area of semicircle  $AcBS_2$  + area semicircle  $AbCS_1$   
 + area triangle  $ABC$  – area semicircle  $aBAC$

$$= \frac{\pi R_2^2}{2} + \frac{\pi R_1^2}{2} + \frac{AC \times AB}{2} - \frac{\pi R_G^2}{2}$$

$$= \frac{\pi}{2} [(AB/2)^2 + (AC/2)^2 - (BC/2)^2] + \frac{AC \times AB}{2}$$

$$= \frac{\pi}{8} [(AB)^2 + (AC)^2 - (BC)^2] + \frac{AC \times AB}{2}$$

but  $(AB)^2 + (AC)^2 = (BC)^2$ , so  $[(AB)^2 + (AC)^2 - (BC)^2] = 0$

and Area  $S_1$  + area  $S_2$  =  $\frac{AC \times AB}{2}$ , the area of the triangle, which is one acre.





# Solution to Problem 54

The area of triangle ABC = 1 acre = 43,560 sq. ft =  $\frac{1}{2} AB \times AC$ ,  
 but  $AB = 0.75 AC$ , so  $(2)(43560) = (0.75 AC)(AC)$  or  $87120 = 0.75AC^2$

from which  $AC = 340.822'$ ,  $AB = 255.617'$  and  $BC = 426.028'$  (By proportions)

$ab = \frac{1}{2} AB = 127.808'$ ,  $Ab = bC = \frac{1}{2} AC = 170.411'$ , and  $Ca = aB = Aa = 213.014'$

$$\cos \text{angle } AaC = \frac{(aA)^2 + (aC)^2 - (AC)^2}{2(aA)(aC)} = \frac{213.014^2 + 213.014^2 - 340.822^2}{(2)(213.014)^2}$$

$$= -0.279996996 \text{ and angle } AaC = 106^\circ 15' 36''$$

Area  $S_1$  = Area semicircle  $AS_1C$  + area triangle  $ACa$  – area sector  $AaC$

$$\text{Area } S_1 = \frac{1}{2} R_1^2 + 127.808 \times 170.411 - (106^\circ 15' 36'' / 360^\circ)(213.014^2) \pi$$

$$= 45,615.925 + 21,780.000 - 42,075.939 = 25,319.986 \text{ sq. ft.}$$

Area  $S_2$  = Area semicircle  $AS_2B$  + area triangle  $AaB$  – area sector  $AaB$

$$\text{Area } S_2 = \frac{1}{2} R_2^2 + 127.808 \times 170.411 - (73^\circ 44' 24'' / 360^\circ)(213.014^2) \pi$$

$$= 25,658.958 + 21,780.000 - 29,198.944 = 18,240.014 \text{ sq. ft.}$$

Area  $S_1$  + area  $S_2$  = 25,319.986 sq. ft. + 18,240.014 sq. ft. = 43,560.000 sq. ft.

Area  $G$  =  $\pi R_G^2 + 43,560 - \pi R_1^2 - \pi R_2^2 + \text{segment } Ab'Q + \text{segment } Ac'Q$

$$= \pi(R_G^2 - R_1^2 - R_2^2) + 43,560 + \text{segment } Ab'Q + \text{segment } Ac'Q$$

$$= 43,560 + (73^\circ 44' 24'' / 360^\circ) \pi (170.411^2) - 136.329 \times 102.247$$

$$+ (106^\circ 15' 36'' / 360^\circ) \pi (127.808^2) - 76.685 \times 102.247$$

$$= 55,614.66 \text{ sq. ft.}$$

