Solution to Problem 49

AREA OF A-B-C = ½ bc sin 36° = ½ ab sin 60° = ½ ac sin 84° = 4,356,,000 sq. ft.

bc = 14,821,739.68 sq. ft., so that b = 14,821,739.68 / c

ab = 10,059,751.09 sq. ft., and by substitution from above

a / c = 14,821,739.68 = 10,059,751.09 sq. ft.,

or 10,059,751.09 c / 14,821,739.68 = a

ac = 8,759,988.132 sq. ft.

10,059,751.09 c^2 / 14,821,739.68 = 8,759,988.132

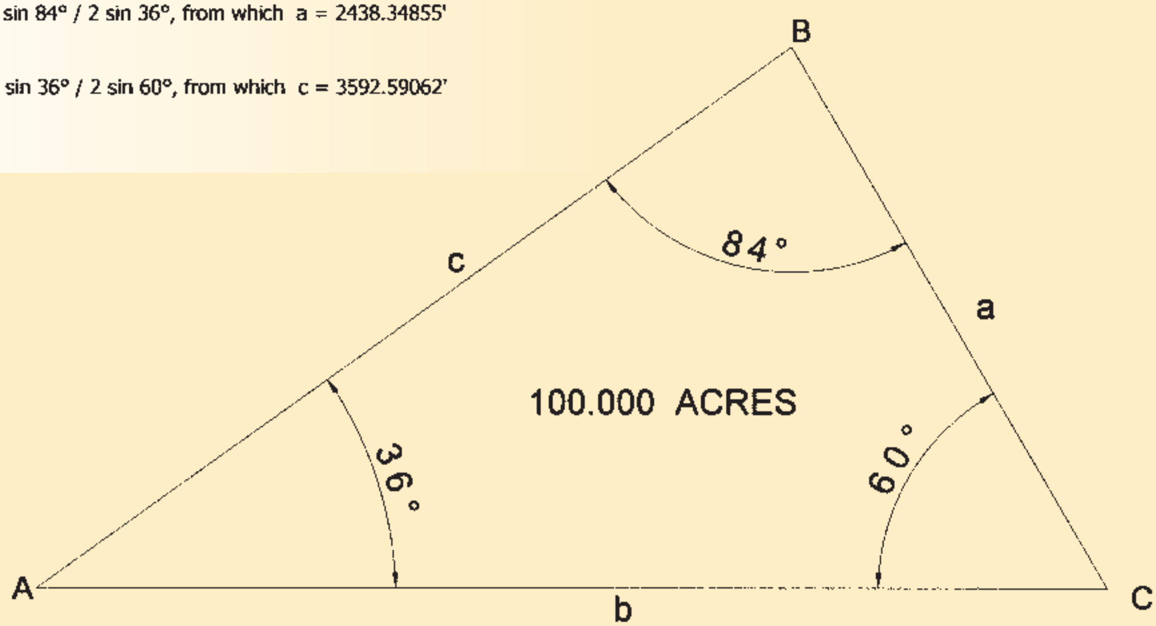
from which c= 3592.5906′, b = 4125.6411′, and a = 2438.3485′

Alternatively:

Area = b^2 sin 36° sin 60° / 2 sin 84°, from which b = 4125.64114'

Area = a^2 sin 60° sin 84° / 2 sin 36°, from which a = 2438.34855'

and Area = c^2 sin 84° sin 36° / 2 sin 60°, from which c = 3592.59062'



Solution to Problem 50

Pick any point, b′, on line A-C (I used the midpoint), construct perpendiculars to A-B and B-C at points c′ and a′ respectively. Calculate the areas of A-c′-b′ (=1,011,740.68 square feet) and C-b′-a′ (=921,283.98 square feet).

Let P be at distance m and angle ϕ as shown.

Area b′-c′-c-P + area P-b-b′ + area A-c′-b′ = 1,452,000 square feet

Area P-a-a′-b′ + area C-b′-a′ – area P-b-b′ = 1,452,000 square feet

Area b′-c′-c-P + area P-b-b′ = $m\cos(54°-\phi)\,(1/2)\,[\,1212.4955+1212.4955 - m\sin(54°-\phi)]$

$+(1/2)\,m^2\sin\phi\cos\phi = 440{,}259.32$, or

$(m\cos54°\cos\phi + m\sin54°\sin\phi)(\,2424.991 - m\sin54°\cos\phi + m\cos54°\sin\phi\,)$

$+ m^2\sin\phi\cos\phi = 880{,}518.64$, which rearranges to

Eq. 1: $m^2(0.475528258\sin^2\phi - 0.475528258\cos^2\phi + 0.690983006\sin\phi\cos\phi) +$

$m(\,1425.37395\cos\phi + 1961.85893\sin\phi) - 880{,}518.64 = 0$

Area P-a-a′-b′ – area P-b-b′ = $m\cos(30°+\phi)(1/2)[1786.4551+1786.4551-m\sin(30°+\phi)]$

$-(1/2)\,m^2\sin\phi\cos\phi = 530{,}716.02$, or

$(m\cos30°\cos\phi - m\sin30°\sin\phi)(3572.9102 - m\sin30°\cos\phi - m\cos30°\sin\phi)$

$-m^2\sin\phi\cos\phi = 1{,}061{,}432.04$, which rearranges to

Eq. 2: $m^2(0.433012702\sin^2\phi - 0.433012702\cos^2\phi - 1.5\sin\phi\cos\phi) +$

$m(\,3094.2310\cos\phi - 1786.4551\sin\phi) - 1{,}061{,}432.04 = 0$

Substituting values for ϕ in Eq. 1 and Eq. 2 and solving for m by the quadratic equation yields values of m shown in Table 1. (You can write a program, use a spreadsheet like I did, or just do a lot of calculating!)

Plotting the first six values in Table 1 yields Figure 1, which shows m to be approximately 495 and ϕ to be about 15.25° (I used 10x10 graph paper).

Plotting values closer to 15.25° (15.0° to 15.5°) yields Figure 2 (which shows how linear the functions are in this range). Using the four values closest to the intersection, 15.2° and 15.3°, equations can be written in the form of y=mx+b (as shown on Figure 2) to solve for the intersection: m=495.866 and ϕ=15.2703977° or 15°16′13.4″. (One further refinement of plotting and equation writing yields m=495.8646 and ϕ=15.27041780 or 15°16′13.5″)

Point P can now be located 2193.419′ along AC from A (130.598′ from b′) and 478.359′ off line AC at a right angle to line AC.

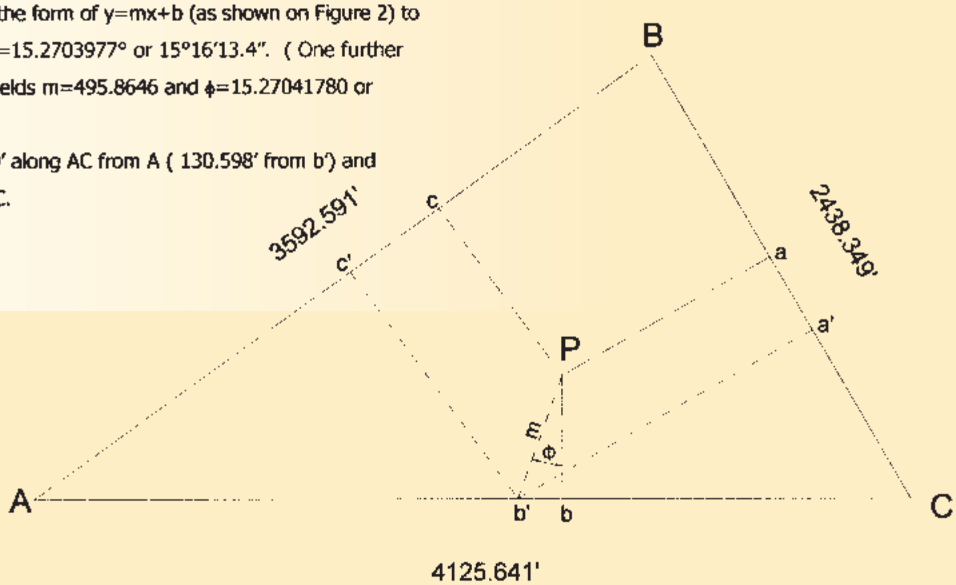


TABLE 1

φ	m from Eq. 1	m from Eq. 2
0°	870.62	361.30
5°	667.97	391.87
10°	564.90	433.32
15°	498.78	491.99
20°	452.46	582.33
25°	418.54	755.45
15.2°	496.6199	494.8463
15.3°	495.5484	496.2942
15.2703977°	495.8648	495.8643
15.270417803°	495.8646	495.8646

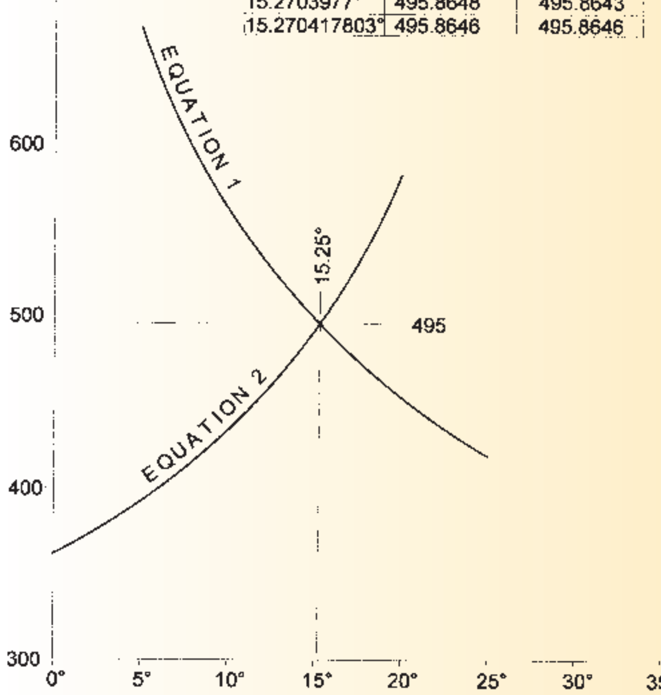


FIGURE 1

FIGURE 2