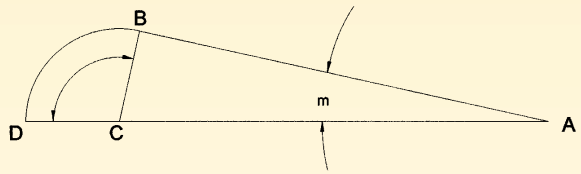
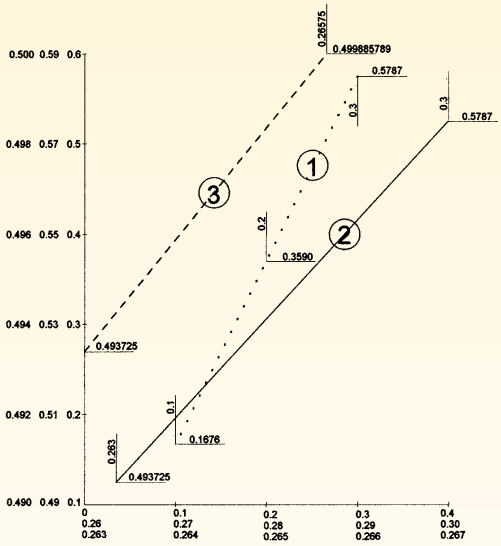
Solution to Problem 40





Let angle BAC be m, so that tan m = CB/AB. But CB = R. If we keep all units in miles, tan m = R. Angle DCB = $\frac{\pi}{2}$ + m, where m is in radians.

Arc BD = R ($\frac{\pi}{2}$ + m) = 0.5 = tan m ($\frac{\pi}{2}$ + m)

Since this is not solvable directly, I used a method I call graphical iteration:

Knowing that 90°>m>0°, or $\pi/2$>m>0 radians, begin by calculating some values for (tan m)($\pi/2$ + m) and see what comes close to 0.5.

Using 0.1 radians yields 0.1676; 0.2 radians yields 0.35896; 0.3 radians yields 0.5787, so the answer must be between 0.2 and 0.3 radians. Plot these values on graph paper (10x10 grid is perfect for this, but any kind will do). I plotted m along the x-axis and the resulting value of (tan m)($\pi/2$ + m) along the y-axis. I used 1"=0.05 m for the x-axis (0 to 0.3) and 1"=0.05 for the y-axis (0.1 to 0.6). Checking opposite the 0.5 value on the y-axis looks like 0.263 on the x-axis. This actually yields a value of 0.493725 for the y-value.

On the same graph sheet,(or another , if you like) but changing the x-axis values to 1"=0.005 m for the range 0.26 to 0.30, and the y-axis to 1"=0.01 for the range 0.49 to 0.58, I replotted 0.3 radians and 0.263 radians. Opposite 0.5 on the y-axis I interpreted 0.26575 radians on the x-axis. Since this actually yields 0.499885789 I tried one more plot.

Using the same graph paper (or another, if you like), I changed the x-axis to be 1"=.0005 m for the range 0.263 to 0.267 and the y-axis to be 1"=0.001 for the range 0.490 to 0.500. I replotted m=0.26575 and m=0.263. Opposite 0.500 on the y-axis is 0.26580 on the x-axis, which actually yields 0.499998033.

The function is linear at this point so I extrapolated from m=0.263 radians, y=0.493725 and m= 0.26575 radians, y=0.499885789 to get m=0.265801 radians.

For m=0.265801 radians, or 15°13'45.4", R=1,437.442'
(Arc BD calculates to be 2,640.002' with this value)